An Introduction to AC Field Hall Effect Measurements

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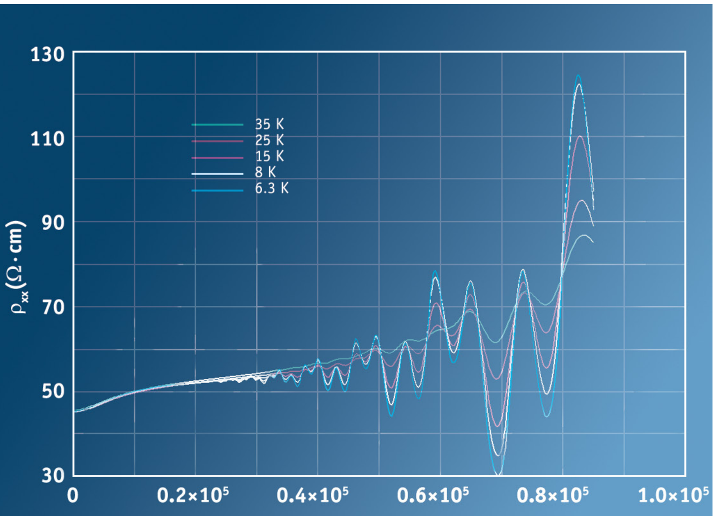
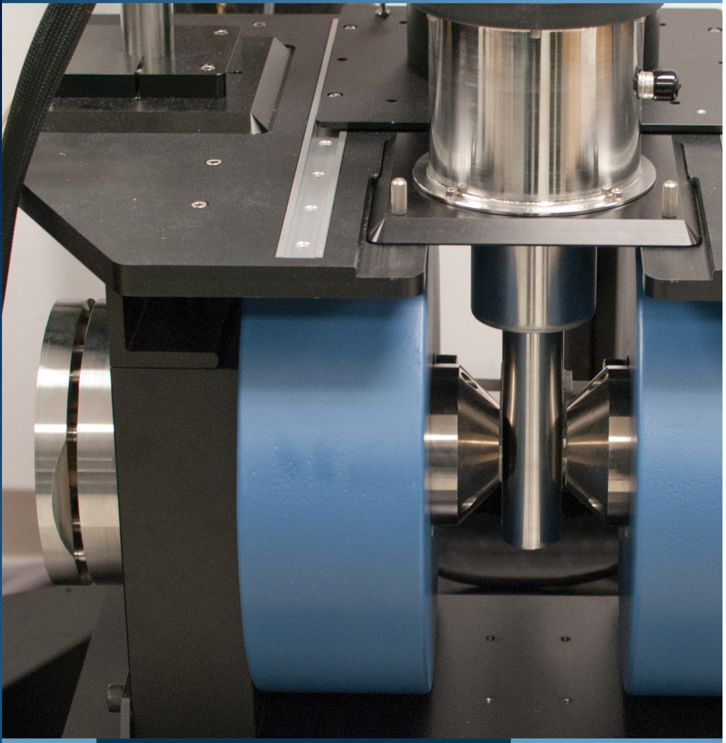
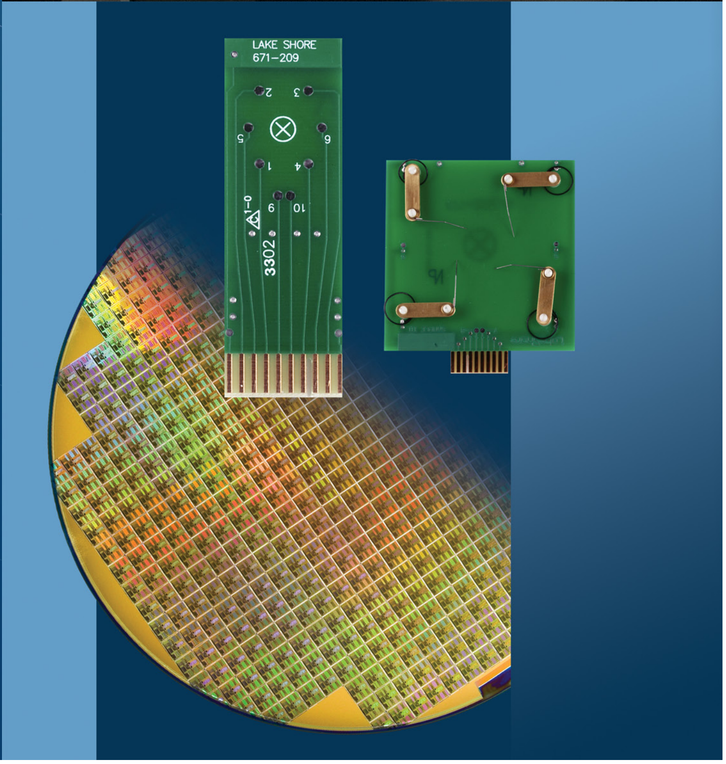
The Hall effect is a well-known method to determine the carrier concentration, carrier type, and when coupled with a resistivity measurement, the mobility of materials. The traditional method used in Hall measurement uses a DC magnetic field. This method has a long history of successful measurements on a wide range of materials including semiconductors. However, materials with low mobility, such as those important in solar cell technology, thermoelectric technology, and organic electronics are very difficult to measure using DC Hall effect methods. This application note will re-introduce a long neglected method using AC fields. Although mentioned in the literature for many years, this technique had little advantage for measuring materials for which the DC measurement provided good results.







Review of DC field Hall measurement protocol

There is a very well developed methodology for measuring the Hall effect and resistivity using DC fields. The methodology is designed to remove the unwanted effects from the measured voltage. The following sections provide a brief summary of this methodology. These explanations are based on the definitions provided here.

The Hall voltage is proportional to the magnetic field (B), current (I), and Hall coefficient (R_H) and depends inversely on the thickness (t). In an ideal geometry the measured Hall voltage is zero with zero applied field. However, the voltage measured in a practical experiment (V_m) also includes a misalignment voltage (V_o) and a thermal electric voltage (V_{TE}). The misalignment voltage is proportional to the resistivity of the material (ρ), the current and a factor (α) that depends on the geometry. This factor converts resistivity to resistance between the two Hall voltage probes. The thermal electric voltage arises from contacts between two different materials and is independent of the current. The thermal electric voltage does depend on the thermal gradients present.

$$V_m = \frac{(R_H iB)}{t} + V_O + V_{TE}$$

$$V_m = \frac{(R_H iB)}{t} + \alpha \frac{\rho}{t} i + V_{TE}$$

The mobility (μ) is the Hall coefficient divided by the resistivity.

$$V_m = \frac{\rho i}{t} (\mu B + \alpha) + V_{TE}$$

The factor α can be as small as zero (for no offset), but typically it is about 1.

Using current reversal to remove the effects of V_{TE}

Current reversal can be used to remove the unwanted effects of thermal electric voltage (V_{TE}). Thermal electric voltage does not depend on current; current reversal exploits this characteristic to remove the effects of V_{TE}. An example of this is shown by measuring the resistance (R) of a sample. The measured voltage, with applied current I_1, can be written as $V_m(I_1) = I_1 R + V_{TE}$. If a second measurement with current I_2 is made, the measured voltage is $V_m(I_2) = I_2 R + V_{TE}$. Normally $I_2 = -I_1$, but allowance is made for cases where I_2 is different from I_1. Then the resistance R can be calculated as $R = (V_m(I_1) - V_m(I_2))/(I_1 - I_2)$. The effect of the thermal electric voltage is removed by subtracting the measured voltage at two different currents.

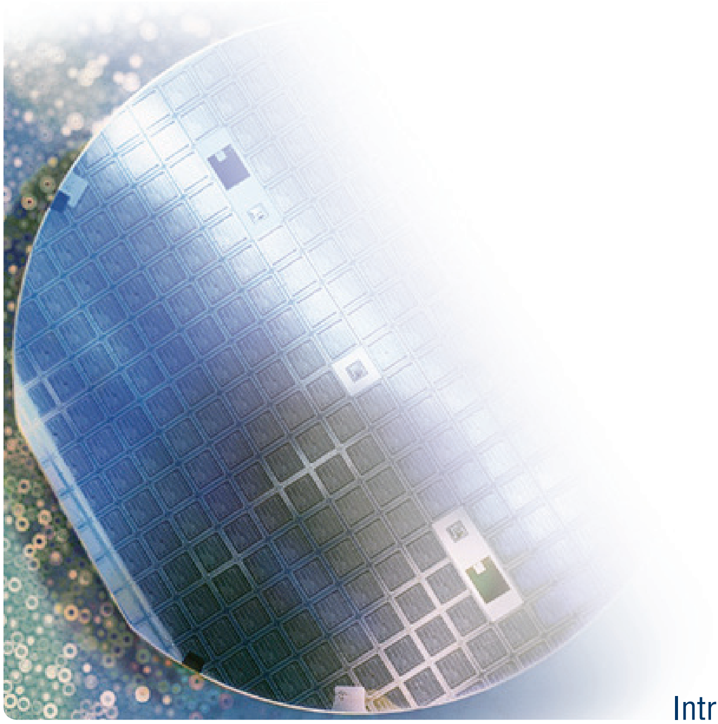
Using field reversal to remove the effects of V_o

Field reversal can be used to remove the unwanted effects of the misalignment voltage. Hall voltage depends on the magnetic field, but the misalignment voltage does not depend on the magnetic field. Assuming that the thermal electric voltages have been removed by current reversal, the measured voltage at a field B_1 can be calculated as $V_m(B_1) = \rho I \mu B_1/t + \rho I \alpha/t$ and the measured voltage at a second field B_2 can be calculated as $V_m(B_2) = \rho I \mu B_2/t + \rho I \alpha/t$. As in current reversal, normally $B_2 = -B_1$. Then the quantity $\rho I \mu/t$ is calculated as $\rho I \mu/t = (V_m(B_1) - V_m(B_2))/(B_1 - B_2)$. Since I and t are known quantities, the Hall coefficient $(R_H = \rho\mu)$ can be obtained. The Hall resistance is defined as $\rho\mu^*(B_1 - B_2) = (V_m(B_1) - V_m(B_2))/I$.

Disadvantages of the DC method

For low mobility materials, the quantity μB can be very small compared to α. When the expression $(V_m(B_1) - V_m(B_2))$ is calculated, the subtraction between the two large numbers gives a small result. Any noise in the measurement can easily dominate the actual quantity, and consequently, produce imprecise results. This is often the reason that Hall measurements on low mobility materials give inconsistent carrier signs.

A second problem is that the two measurements $Vm\ (B_1)$ and $Vm\ (B_2)$ can be separated in time by a significant amount. The time to reverse the field of a magnet can vary from seconds to minutes depending on the magnet configuration. The misalignment voltage $V_0 = \rho I \alpha / t$ depends on the resistivity of the material. If the material changes temperature between the two measurements $V_m(B_1)$ and $V_m(B_2)$, the misalignment voltage will change, and the subtraction will not cancel the misalignment voltage. The un-canceled misalignment voltage will be included in the calculation of the Hall coefficient.



AC Field Hall measurements

A second method to remove the effect of the misalignment is to use an AC magnetic field. If the magnetic field is made a sinusoidal signal $(B(t) = B\sin(\omega t))$, then in the quasi-static approximation, the Hall voltage will become time dependent as well, $V_H(t) = i\,\rho\mu/t\,B\sin(\omega t)$. The misalignment voltage is independent of the magnetic field, and consequently remains a DC voltage. The measured voltage is now

$$V_m = \frac{\rho i}{t}(\mu B \sin(\omega t) + \alpha)$$

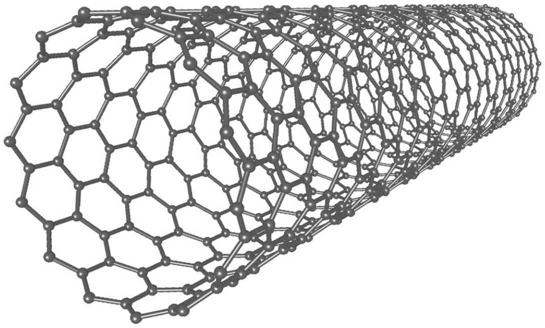
The measurement electronics using a lock-in amplifier can separate the desired AC signal from the undesired DC signal with a high degree of precision. However, there is a new term in the measured voltage. This is proportional to the time derivative of the magnetic field, and it is proportional to the inductance of the sample and the leads used in the measurement. If the proportionality constant is β, the measured voltage should be written as

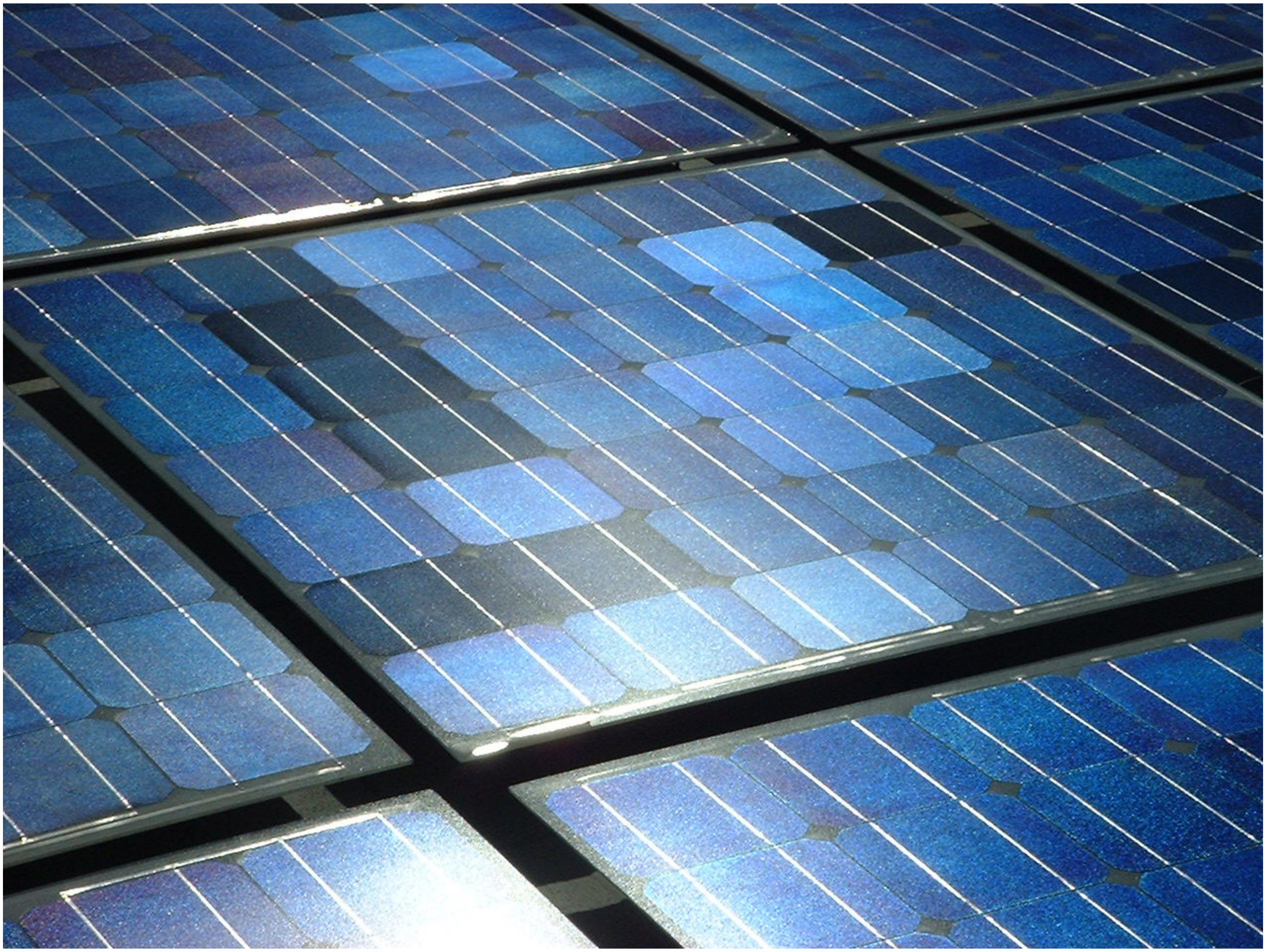
$$V_m = \frac{\rho i}{t}(\mu B \sin(\omega t) + \alpha) + \beta \frac{dB}{dt}$$

Since this is an AC signal, the lock-in will measure this term as well as the Hall voltage term. Since this term is independent of the current, just like the thermal electric voltage, one method is to use current reversal to remove this term. This term is also 90° out of phase from the signal. Phase resolution on the lock-in amplifier can eliminate this term using a combination of current reversal and phase measurement.

Some example measurements

The following two sections show experiments on two different kinds of materials. In order to best explain AC measurements on low mobility samples, a typical, moderately low mobility oxide was chosen for the first example. A comparison between the DC and AC techniques is provided with an explanation of its data. Using this same process, the second example advances into a more difficult material to measure—a micro-crystalline silicon sample of n type with high resistance, and an expected mobility of <1 cm^2/V s.





Transparent oxides

Transparent oxides are important materials for oxide electronics and solar cell applications. We measured a typical, moderately low mobility oxide using both the DC method and the AC method. In this comparison, the material was recorded with a 0.3 T field for the DC technique, and 0.339 T field for the AC technique. For materials with a single carrier, the Hall voltage is a linear function of applied field strength; therefore they do not need to be measured at the same applied field strength.

The details of the DC measurements on this sample are summarized in Table 1. This table shows both the current reversal measurement and the field reversal measurement. The voltage measured at positive current is very nearly the same as the voltage measured at negative current. Using the current reversal equations discussed above, thermal electric voltages in this material are found to be very small (about 5 mV) compared to the approximate 0.030 V measured. However, the current reversed Hall resistance for positive field is 313 Ω and the current reversed Hall resistance for negative field is 324 Ω. The Hall resistance is then calculated using the field reversal equations, and is found to be -5.58 Ω, about 2% of the measured value.

The AC measurement is summarized in Table 2. In this case, the measured voltage is only the Hall voltage, since the misalignment voltage is removed with the measurement electronics. The voltage with positive and negative current is very similar, indicating that the phase error is very small. In this table the voltages are slightly higher than the DC case because the field is slightly different (0.3 T compared to 0.339 T). The Hall coefficient in the AC measurement is very similar to that in the DC measurement. Using the measured DC resistivity of 15217 Ω/sqr and the Hall coefficient, the mobility of the sample is 12.2 $cm^2/(V s)$.

Table 1 DC measurement of a moderately low mobility oxide

	Field (T)	Current (A)	Voltage (V)	V/I (Ω)	Hall resistance with current reversal (Ω)	Hall resistance with field reversal (Ω)	Hall coefficient (m^2/C)
Positive field	0.3	1.00E-04	3.13E-02	3.13E+02	3.13E+02	-5.58E+00	-1.86E+01
Positive field and current reversal		-1.00E-04	-3.13E-02	3.13E+02			
Negative field	-0.3	1.00E-04	3.25E-02	3.24E+02	3.24E+02		
Negative field and current reversal		-1.00E-04	-3.24E-02	3.24E+02			

Table 2 AC measurement of a moderately low mobility oxide

Field (T)	Current (A)	Voltage (V)	V/I (Ω)	Hall resistance (Ω)	Hall coefficient (m^2/C)
0.339	1.00E-04	-6.288E-04	-6.29E+00	-6.29E+00	-1.85E+01
	-1.00E-04	6.281E-04	-6.28E+00		

Micro-crystalline silicon

For this example, measurements were made on a high resistance, 247 MΩ/sqr micro-crystalline silicon sample of n type, with an expected mobility of <1 cm^2/V s. This measurement was conducted with a 0.6 T field for the DC measurement, and a 0.339 T field for the AC measurement. To extract low mobility carriers using DC techniques, very large fields have to be applied to measure the small Hall voltages, whereas in the AC technique this is not necessary, since all the unwanted voltages that contribute to the total measured voltage are easily negated.

The details of the DC measurements on this sample are summarized in Table 3. This table shows both the current reversal measurement and the field reversal measurement. The voltage measured at plus current is different from the voltage measured at negative current. The thermal electric voltages are not small compared to the approximate 1.8 V measured (~70 mV). The current reversed Hall resistance for positive field is 18.5 MΩ and for negative field 18.7 MΩ. The Hall resistance calculated using field reversal is 107 kΩ, about 0.5% of the measured value. The measurement of the Hall coefficient and resistivity give a mobility of 7 cm^2/(V s), which is much larger than the expected value.

The AC measurement is summarized in Table 4. In this case the measured voltage is only the Hall voltage, since the misalignment voltage is removed before the measurement. However, the current reversed voltages differ from each other. This means the phase errors for the high resistance sample are large. Using the quadrature signal as well as the in phase signal, the Hall resistance of is corrected for the phase error to 970 . Note that the Hall resistance measured with the AC method is approximately 1% of the DC Hall resistance. The mobility for this measurement is 0.1 cm^2/(V s), more in line with the expected results.

Table 1 DC measurement of a high resistance micro-crystalline silicon sample of n type

	Field (T)	Current (A)	Voltage (V)	V/I (Ω)	Hall resistance with current reversal (Ω)	Hall resistance with field reversal (Ω)	Hall coefficient (m^2/C)
Positive field	0.6	9.93E-08	1.91E-00	1.92E+07	—	-1.07E+05	-1.78E+05
Positive field and current reversal		-9.92E-08	-1.77E-00	1.78E+07	1.85E+07		
Negative field	-0.6	9.93E-08	1.92E-00	1.93E+07	—		
Negative field and current reversal		-9.92E-08	-1.80E-00	1.81E+07	1.87E+07		

Table 2 AC measurement of a high resistance micro-crystalline silicon sample of n type

	Field (T)	Current (A)	Voltage (V)	V/I (Ω)	Hall resistance (Ω)	Hall coefficient (m^2/C)
In phase signal	0.339	1.00E-07	-1.212E-04	1.21E+03	-9.70E+02	-2.86E+03
		-1.00E-07	2.219E-04	2.22E+03		
Quadrature signal	0.339	1.00E-07	1.212E-04	1.21E+03	—	—
		-1.00E-07	-6.080E-05	1.051E-04		

Conclusion

The samples explored for this document were chosen specifically to provide transparency concerning the validity of the AC Hall effect. The first sample of moderately low mobility material would be easily recognized and tested in many laboratories. By looking at the mobilities derived in the transparent oxide sample using the DC Hall effect first, and then the AC Hall effect, you can see that the AC Hall effect provided results comparable to the DC Hall effect method. The Hall coefficient in the AC measurement is very similar to that in the DC measurement. Using this calculation along with the measured DC resistivity of 15217 Ω/sqr, the mobility of the sample was found to be ~12.2 cm^2/(V s). The second set of examples then explored the more difficult sample for which it is known that the DC method is not adequate. While the sample's mobility was expected to be <1 cm^2/V s , the DC method provided a result much higher than that (7 cm^2/(V s)). However, the AC method, with its ability to derive measurements of high resistance samples, measured the resistance at 1% of that measured with the DC method. This resulted in obtaining the low mobility for this sample of 0.1 cm^2/(V s).

With the capability to measure low mobility materials, the AC Hall effect method provides solutions to those exploring important materials used in solar cell, thermoelectric, and organic materials.

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